Vacuum effects of an ultralow mass particle account for the recent acceleration of the universe

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In recent work, we showed that non-perturbative vacuum effects of a very low mass particle could induce, at a redshift of order 1, a transition from a matter-dominated to an accelerating universe. In that work, we used the simplification of a sudden transition out of the matter-dominated stage and were able to fit the type Ia supernovae (SNe-Ia) data points with a spatially open universe. In the present work, we find a more accurate, smooth *spatially-flat* analytic solution to the quantum-corrected Einstein equations. This solution gives a good fit to the SNe-Ia data with a particle mass parameter m_h in the range 6.40×10^{-33} eV to 7.25×10^{-33} eV. It follows that the ratio of total matter density (including dark matter) to critical density, Ω_0, is in the range 0.58 to 0.15, and the age t_0 of the universe is in the range $8.10h^{-1}$ to $12.2h^{-1}$ Gyr, where h is the present value of the Hubble constant, measured as a fraction of the value 100 km/(s Mpc). This spatially-flat model agrees with estimates of the position of the first acoustic peak in the small angular scale fluctuations of the cosmic background radiation and with light-element abundances of standard big-bang nucleosynthesis. Our model has only a single free parameter, m_h, and does not require that we live at a special time in the evolution of the universe. [S0556-2821(99)01024-3]

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I. INTRODUCTION

Many attempts [1–3] have been made to account for the unexpected behavior of the recent expansion of the universe. In a previous paper [4], we showed that non-perturbative quantum effects in the vacuum may account for the supernovae type Ia (SNe-Ia) data [5,6], suggesting a recent acceleration of the expansion of the universe. We considered a free quantized scalar field (with possible coupling to the scalar curvature, R). The propagator of the field included an infinite sum of all terms having at least one factor of R [7,8]. The effective action was given in [4], which can be referred to by the reader for a more complete exposition of the theory. We found an approximate solution to the effective Einstein gravitational field equations. In that solution, there was a sudden transition (at redshift z of order 1) from an earlier matter dominated stage of the expansion to a mildly inflating de Sitter expansion. This sudden transition model required the universe to be spatially open in order to fit the cosmological data.

Here we find an improved analytic solution to the effective Einstein equations and determine its consequences. Surprisingly, we find that the analytic solution permits the SNe-Ia data to be fit with a spatially flat model having a reasonable matter density and age. The spatial flatness makes a significant difference [9] in attempting to fit the spectrum of small angular scale fluctuations of the cosmic microwave background radiation (CMBR). It appears that the existing data favor a spatially flat cosmological model. Spatial flatness is also favored in models of the universe having an early inflationary stage.

In our present model, there is only one free parameter,

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namely, the ratio of the mass parameter $\bar{m}$ to the present value of the Hubble constant, H_0. It is convenient to express this parameter in the form

$$m_h \equiv \bar{m}/h, \tag{1}$$

where

$$h \equiv H_0/[100 \text{ km/(s Mpc)}], \tag{2}$$

with H_0 expressed in km/(s Mpc). We find that a good fit to the SNe-Ia data is obtained when m_h is in the range 6.40×10^{-33} eV $< m_h < 7.25 \times 10^{-33}$ eV. For this parameter range, the ratio of the total matter density (including dark matter) to critical density, Ω_0, is found to be in the range $0.58 > \Omega_0 > 0.15$, and the age t_0 of the universe is found to be in the range $8.10h^{-1}$ Gyr $< t_0 < 12.2h^{-1}$ Gyr. Also, we find that our model gives reasonable abundances for light elements formed during big-bang nucleosynthesis. In our model, no special coincidence at the present time is necessary to explain why the energy density of matter is of the same order as the vacuum energy density.

The organization of this paper is as follows. In Sec. II, we give the effective Einstein equation of the model, and summarize the solution of Ref. [4]. In Sec. III, we derive an analytic solution for the scale factor of the model. In Sec. IV, we compare the model to SNe-Ia data and obtain ranges for m_h, Ω_0 and t_0. In Sec. V, we discuss the implications of the model for big-bang nucleosynthesis and for the spectrum of CMBR fluctuations. In Sec. VI, we show it is probable (independent of fitting the observations) that, at the present time, the vacuum energy density in our model is comparable to the matter density. Finally, our conclusions are given in Sec. VII.

II. OUR MODEL

In Ref. [4], we consider a free, massive quantized scalar field of inverse Compton wavelength (or mass) m and curvature coupling ξ. The effective action for gravity coupled to such a field is obtained by integrating out the vacuum fluctuations of the field [4,7]. This effective action is the simplest one that gives the standard trace anomaly in the massless-conformally-coupled limit, and contains the explicit sum (in arbitrary dimensions) of all terms in the propagator having at least one factor of the scalar curvature, R.

The trace of the Einstein equations obtained by variation of this effective action with respect to the metric tensor takes the following form in a Friedmann-Robertson-Walker (FRW) spacetime (in units such that $c=1$):

$$R + \frac{T_{cl}}{2\kappa_0} - 4\Lambda_0 = \frac{\hbar m^2}{32\pi^2 \kappa_0} \left\{ (m^2 + \bar{\xi}R)\ln|1 + \bar{\xi}Rm^{-2}| \right.$$
$$- \frac{m^2 \bar{\xi} R}{m^2 + \bar{\xi}R} \left(1 + \frac{3}{2}\bar{\xi}\frac{R}{m^2} + \frac{1}{2}\bar{\xi}^2\frac{R^2}{m^4} \right)$$
$$\left. \times [\bar{\xi}^2 - (1080)^{-1}] + v \right) \right\}, \quad (3)$$

where T_{cl} is the trace of the stress tensor of a classical, perfect fluid component containing mixed matter and radiation, Λ_0 is the cosmological constant, $\kappa_0 = (16\pi G)^{-1}$ (G is Newton's constant), $\bar{\xi} = \xi - 1/6$, and v is a quantity that vanishes in de Sitter space:

$$v = \frac{1}{180m^4} \left(\frac{1}{4}R^2 - R_{\mu\nu}R^{\mu\nu} \right). \quad (4)$$

Here, the metric is

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (5)$$

where $a(t)$ is the scale factor and $k = 0, 1, -1$ give spatially flat, closed and open universes respectively.

The right-hand side (RHS) of Eq. (3) is proportional to the trace of the quantum contribution to the stress tensor. In Eq. (3), we have assumed that terms in the quantum stress tensor that depend on derivatives of the curvature are negligible. This assumption is consistent with the solution that we obtain.

In [4], we found that Eq. (3) admits a solution in which a matter-dominated FRW universe transits to a stage of the expansion in which the scalar curvature is nearly constant. This led us to construct an approximate cosmological model which makes a *sudden* transition to a de Sitter universe out of a matter-dominated one. In that approximation, we needed a spatially open cosmology in order to fit high-redshift SNe-Ia data, and to get a reasonable value of the present matter density.

In this paper, we eliminate the assumption of a sudden transition to a de Sitter expansion, by finding an analytic solution to the effective Einstein equations of our model. Consistency with cosmological observations requires that this improved analytic solution have zero (or nearly zero) spatial curvature. This solution consists of a matter-dominated universe smoothly, with continous first and second derivatives of the scale factor (i.e., C^2), joined to a constant scalar curvature expansion that asymptotes to a de Sitter universe at late times. The spatial flatness is consistent with estimates of the first acoustic peak of small angular scale fluctuations in the CMBR, which suggest that the universe is spatially flat (although this issue awaits an ultimate decision at the time of writing; see Ref. [10] for details).

We now turn to the derivation of a smooth solution for the cosmological scale factor, $a(t)$.

III. ANALYTIC SOLUTION

In the following analysis, as in [4], we take $\Lambda_0 = 0$. For $\bar{\xi} < 0$, and with a low value of $\bar{m}$ ($\simeq 10^{-33}$ eV), we find in [4] that the evolution of the FRW universe is essentially unaffected by the quantum contributions at early times. A perturbative analysis in $\hbar$ shows that quantum contributions to the stress tensor begin to have a significant effect at a time t_j, when the classical scalar curvature has decreased to a value given roughly by

$$R_{cl}(t_j) \equiv \frac{\rho_m(t_j)}{2\kappa_0} = \bar{m}^2, \quad (6)$$

where

$$\bar{m}^2 \equiv m^2/(-\bar{\xi}). \quad (7)$$

This effect is shown to occur well into the matter-dominated stage of the evolution (therefore, for $t > t_j$, $T_{cl} \simeq -\rho_m$). We further argue that, after the scalar curvature reaches the value R_{cl} (i.e. for $t \geq t_j$), it stays essentially constant during the later evolution of the universe. More precisely, for all times $t > t_j$, we find that the scalar curvature has the form

$$R(t) = \bar{m}^2[1 - \epsilon(t)], \quad (8)$$

where

$$\epsilon(t) = \frac{1}{2}\bar{\xi}[-\delta + (\delta^2 + 4\beta\bar{\xi}^{-1})^{1/2}] \quad (9)$$

and

$$\delta = -\frac{\rho_m(t)}{2m^2\kappa_0} - \bar{\xi}^{-1}, \quad \beta = \frac{r}{2\pi}\left(v(t) - \frac{1}{2160\bar{\xi}^2} \right). \quad (10)$$

Here,

$$r = \frac{m^2}{m_{Pl}^2}, \quad (11)$$

m_{Pl} being the Planck mass. As shown in [4], it is straightforward to verify that $\epsilon(t)$ is of order r for $t > t_j$ and of order

$\sqrt{r}$ at $t=t_j$. For the low value of r under consideration, it follows that $\epsilon(t)\ll 1$ for all $t\geq t_j$. Thus the scalar curvature stays nearly constant at the value $\bar{m}^2$ for $t\geq t_j$.

Despite the constancy of the scalar curvature, the quantum contribution to the stress tensor increases dramatically from the time t_j to the present time t_0. To see this, consider the trace of the quantum stress tensor, T_q, defined by $T_q/(2\kappa_0)=-[$RHS of Eq. (3)$]$. It follows from Eq. (3) that (with $\Lambda_0=0$)

$$T_q=-2\kappa_0 R-T_{cl}$$
$$=-2\kappa_0\bar{m}^2+\rho_m+\mathcal{O}(\epsilon), \qquad (12)$$

where the second approximate equality holds for $t\geq t_j$. Assuming (as we find) that the transition time t_j occurs at a redshift z_j of order 1, and given that $\rho_m(t_j)\simeq 2\kappa_0\bar{m}^2$ [from Eq. (6)], we obtain the matter density at the present time, $\rho_m(t_0)$, as

$$\rho_m(t_0)=-T_{cl}(t_0)=[a(t_j)/a(t_0)]^3\rho_m(t_j)$$
$$\simeq(1+z_j)^{-3}2\kappa_0\bar{m}^2$$
$$\simeq\kappa_0\bar{m}^2/4+\mathcal{O}(\epsilon). \qquad (13)$$

Thus, according to Eq. (12), T_q grows from $T_q(t_j)$ $\simeq-2\kappa_0\bar{m}^2+2\kappa_0\bar{m}^2\simeq 0$ at time t_j to $T_q(t_0)\simeq-2\kappa_0\bar{m}^2$ $+(1/4)\kappa_0\bar{m}^2\simeq-(7/4)\kappa_0\bar{m}^2\simeq 7T_{cl}(t_0)$ at the present time t_0. Thus, we find that the quantum contribution to the stress tensor grows from a negligible value at a redshift of order 1 to a value exceeding the classical contribution at the present time.

To obtain the scale factor $a(t)$, we consider a spatially flat ($k=0$) cosmology, and find the constant scalar curvature solution for $t\geq t_j$ that joins to the usual matter-dominated solution at time t_j, with continous first and second derivatives. The time t_j obtained from Eq. (6), using $R_{cl}(t)$ $=(4/3)t^{-2}$ for $t\leq t_j$ in the spatially flat matter-dominated universe, is

$$t_j=(2/\sqrt{3})\bar{m}^{-1}. \qquad (14)$$

For such a universe the Hubble constant $H(t)=(2/3)t^{-1}$ has the value at t_j given by

$$H(t_j)=\bar{m}/\sqrt{3}. \qquad (15)$$

For $t>t_j$, we now obtain the unique solution to the constant scalar curvature equation

$$R=6\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}\right)=\bar{m}^2, \qquad (16)$$

which satisfies

$$H(t_j)=\dot{a}(t_j)/a(t_j)=\bar{m}/\sqrt{3}. \qquad (17)$$

Continuity of the scalar curvature at t_j ensures that the second derivative of the scale factor is also continuous. This solution is

$$a(t)=a(t_j)\sqrt{\sinh\left(\frac{t\bar{m}}{\sqrt{3}}-\alpha\right)\Bigg/\sinh\left(\frac{2}{3}-\alpha\right)}, \quad t>t_j, \qquad (18)$$

where

$$\alpha=\frac{2}{3}-\tanh^{-1}\left(\frac{1}{2}\right)\simeq 0.117. \qquad (19)$$

We have verified that this solution also satisfies the remaining Einstein equations up to terms of order ϵ. According to Eq. (18), the expansion approaches a de Sitter expansion at late times (i.e., $t\bar{m}\gg 1$). Furthermore, it has the property that the deceleration parameter $q_0\equiv-\ddot{a}a/\dot{a}^2$ is negative (i.e., the universe is accelerating) for all $t>t_a$, where

$$t_a=\sqrt{3}\bar{m}^{-1}[\alpha+\tanh^{-1}(2^{-1/2})]\simeq 1.73\bar{m}^{-1}\simeq 1.50t_j. \qquad (20)$$

Also, the solution joins in a smooth (C^2) manner to the usual spatially flat matter-dominated solution for $t<t_j$, i.e.

$$a(t)=a(t_j)(t/t_j)^{2/3}=(\sqrt{3}\bar{m}t/2)^{2/3}, \quad t<t_j. \qquad (21)$$

This solution, given by Eqs. (18), (19) and (21), depends on only one parameter, $\bar{m}$. It is more accurate than the approximation of a sudden transition to a de Sitter expansion, that we used in [4]. Our analytic solution takes a long time (of order $\bar{m}^{-1}$) to approach a de Sitter expansion. This is the reason that the spatially flat model gives agreement with observations (as we show below). In the approximation of a sudden transition to a de Sitter expansion, consistency with observations required a negative spatial curvature. We further note that our analytic solution is not the same as that of a mixed matter–cosmological-constant model, for which $a(t)\propto[\sinh(\sqrt{3\Lambda}t/2)]^{2/3}$.

Further improvements in the accuracy of our solution can be made by an iterative procedure in which the present solution is used to evaluate $\epsilon(t)$ in Eqs. (9) and (10), and then Eq. (16) is solved with $\bar{m}^2$ replaced by the RHS of Eq. (8).

IV. COMPARISON WITH OBSERVATIONS

For comparison with observations of high-redshift type 1a supernovas (SNe-Ia), we calculate the luminosity distance-redshift relation for the model defined by Eqs. (18) and (21), and, from it, the difference in apparent and absolute magnitudes as a function of the redshift z of a source. This difference is given by

$$m-M=5\log_{10}d_L+25, \qquad (22)$$

where d_L is the luminosity distance to the source in Mpc, defined as [11]

$$d_L = (1+z)a_0 r_1, \qquad (23)$$

where a_0 is the present scale factor, and r_1 is the comoving coordinate distance from a source at redshift z to a detector at redshift 0. For a spatially flat FRW universe, r_1 is given by

$$r_1 = ca_0^{-1} \int_0^z \frac{dz'}{H(z')}, \qquad (24)$$

where $H(z)$ is the Hubble constant as a function of z, and the speed of light c is now shown explicitly. It is convenient to introduce the parameter $h \equiv H_0/[100 \text{ km/(s Mpc)}]$, where H_0 is the present value of the Hubble constant in km/(s Mpc). It is also useful to work with the rescaled mass parameter m_h of Eq. (1). We note that m_h, being an inverse length, can be measured in units of Mpc^{-1}.

For our model, we define z_j as the redshift at time t_j. Then we obtain, using Eqs. (18) and (24),

$$hr_{1<}(z) = a_0^{-1} \sqrt{12} m_h^{-1} \chi \int_1^{1+z} \frac{dx}{\sqrt{x^4 + \chi^2}}, \qquad (25)$$

where $r_{1<}(z)$ denotes $r_1(z)$ for $z < z_j$, and

$$\chi = \sinh\left(\frac{cht_0 m_h}{\sqrt{3}} - \alpha\right). \qquad (26)$$

For $z > z_j$, we use Eqs. (21) and (24) to obtain

$$hr_{1>}(z) = hr_{1<}(z_j) + 2\sqrt{3} a_0^{-1} m_h^{-1}(1+z_j)^{3/2}$$
$$\times [(1+z_j)^{-1/2} - (1+z)^{-1/2}]. \qquad (27)$$

Equation (23) gives the luminosity distance for this model, $d_{L1}(z)$, defined by

$$d_{L1}(z) = (1+z)a_0 r_{1<}(z) \quad (z < z_j)$$
$$= (1+z)a_0 r_{1>}(z) \quad (z > z_j). \qquad (28)$$

After substituting for $r_{1<}(z)$ and $r_{1>}(z)$ in the above equations, $hd_{L1}(z)$ does not depend on a_0 (for a spatially flat universe). The three parameters that occur in hd_{L1} are ht_0, z_j and m_h. However, we may differentiate Eq. (18) to obtain

$$H(t) = \frac{c\bar{m}}{\sqrt{12}} \coth\left(\frac{ct\bar{m}}{\sqrt{3}} - \alpha\right). \qquad (29)$$

The above equation, evaluated at the present cosmic time t_0, can be rewritten in the form

$$ht_0 = (3.26 \times 10^6 \text{ yr/Mpc}) m_h^{-1}$$
$$\times \{\tanh^{-1}[(865.4 \text{ Mpc}) m_h] + \alpha\}, \qquad (30)$$

which gives ht_0 in years. Also, Eq. (18) yields

$$1+z_j \equiv a(t_0)/a(t_j)$$
$$= \sqrt{\sinh\left(\frac{cht_0 m_h}{\sqrt{3}} - \alpha\right) \Big/ \sinh\left(\frac{2}{3} - \alpha\right)}. \qquad (31)$$

Equation (30) shows that ht_0 is a function of m_h alone. Therefore, the redshift z_j as given by Eq. (31) is also a function of m_h alone. This implies that the function $hd_{L1}(z)$ depends on a single parameter, m_h. The present ratio of the matter density to critical density, Ω_0, is also a function of the same parameter. To see this, we use $\rho_m \propto a^{-3}$. Then one has

$$\Omega_0 \equiv \frac{8\pi G}{3H_0^2} \rho_m(t_0) = \frac{8\pi G}{3H(t_j)^2} \rho_m(t_j) \left(\frac{H(t_j)}{H_0}\right)^2 \left(\frac{a(t_j)}{a_0}\right)^3. \qquad (32)$$

Continuity with the spatially flat matter-dominated universe at $t = t_j$ requires that $[8\pi G \rho_m(t_j)]/[3H(t_j)^2] = 1$. Therefore

$$\Omega_0 = [H(t_j)/H_0]^2 [a(t_j)/a_0]^3$$
$$= (2.996 \times 10^6 \text{ Mpc}^2) m_h^2$$
$$\times \left[\sinh\left(\frac{cht_0 m_h}{\sqrt{3}} - \alpha\right) \Big/ \sinh\left(\frac{2}{3} - \alpha\right)\right]^{-3/2}, \qquad (33)$$

where we have used Eqs. (15) and (18) in arriving at the last equality. Note that the quantity ht_0 appearing in the above equation is itself a function of m_h [see Eq. (30)]. Therefore Ω_0 is also a function of the parameter m_h, which is the only adjustable parameter in our model.

If the time t_j is less than the present age t_0, the monotonic behavior of Eq. (29) implies that $H(\infty) < H_0 < H(t_j)$. With $H(\infty)$ and $H(t_j)$ obtained from Eq. (29), the previous inequality gives

$$\bar{m} > \sqrt{3} H_0/c = 5.78 \times 10^{-4} h \text{ Mpc}^{-1}, \qquad (34)$$

$$\bar{m} < \sqrt{12} H_0/c = 1.16 \times 10^{-3} h \text{ Mpc}^{-1}. \qquad (35)$$

We therefore find that the rescaled mass parameter m_h is constrained by the model to lie in the range

$$5.78 \times 10^{-4} \text{ Mpc}^{-1} < m_h < 1.16 \times 10^{-3} \text{ Mpc}^{-1}. \qquad (36)$$

The above range on m_h can be also expressed in electron volts (eV), as

$$3.69 \times 10^{-33} \text{ eV} < m_h < 7.39 \times 10^{-33} \text{ eV}. \qquad (37)$$

We emphasize that the above range of values of m_h is a *consequence* of our model, if $t_0 > t_j$, independent of any fit to cosmological observations.

Plots of Ω_0 and ht_0 vs m_h, using Eqs. (33) and (30) respectively, are given in Fig. 1, for the range of m_h values of Eq. (37) above. For the same range of m_h values, Fig. 2 is a plot of Ω_0 vs ht_0.
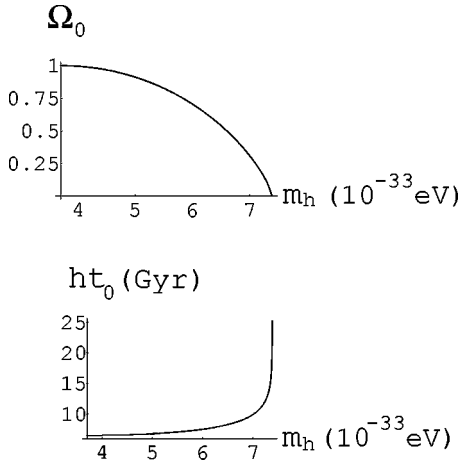
FIG. 1. Plots of Ω_0 and ht_0 versus m_h, with the range of m_h values 3.69×10^{-33} eV$<m_h<7.40\times10^{-33}$ eV.

We now use the luminosity distance of Eq. (28) to fit the SNe-Ia data. To do so, it is convenient to normalize the difference $m-M$ of Eq. (22) to its value in an open universe with $\Omega_0=0.2$. We define

$$\Delta(m-M)(z)\equiv5\log_{10}\left(\frac{d_{L1}(z)}{d_{L2}(0.2,z)}\right)=5\log_{10}\left(\frac{hd_{L1}(z)}{hd_{L2}(0.2,z)}\right),$$

$$(38)$$

where

$$d_{L2}(\Omega_0,z)=2H_0^{-1}c\Omega_0^{-2}[\Omega_0z+(\Omega_0-2)(\sqrt{1+\Omega_0z}-1)]$$

$$=h^{-1}(5995.8\ \text{Mpc})\Omega_0^{-2}$$

$$\times[\Omega_0z+(\Omega_0-2)(\sqrt{1+\Omega_0z}-1)].\qquad(39)$$

Since $hd_{L1}(z)$ is a function of m_h and z, and $hd_{L2}(0.2,z)$ is a function of z alone, it follows that $\Delta(m-M)$ is a function of m_h and z.

Figure 3 is a plot of $\Delta(m-M)$ vs z, along with a plot of SNe-Ia data acquired from Ref. [5]. A good fit is obtained by any curve that lies in between the two dashed curves shown, where curve (a) has $m_h=6.40\times10^{-33}$ eV, and curve (b) has $m_h=7.25\times10^{-33}$ eV. This gives a value of Ω_0 in the range

$$0.15<\Omega_0<0.58\qquad(40)$$

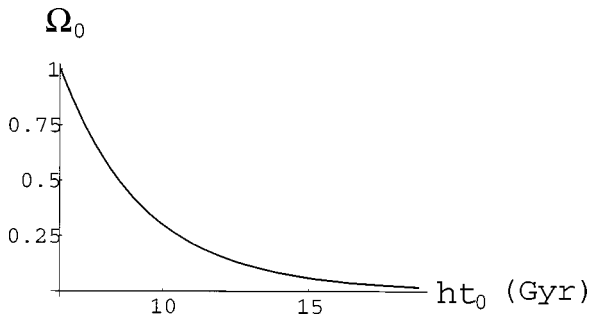FIG. 2. A plot of Ω_0 versus ht_0, with the range of m_h values 3.69×10^{-33} eV$<m_h<7.40\times10^{-33}$ eV.
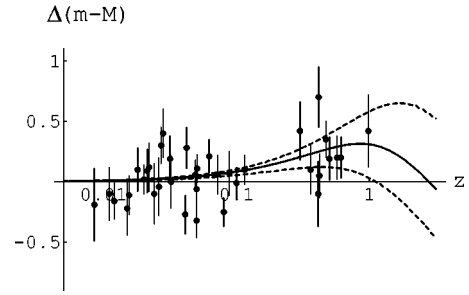

FIG. 3. A plot of the difference between apparent and absolute magnitudes, as functions of redshift z, normalized to an open universe with $\Omega_0=0.2$ and zero cosmological constant. The points with vertical error bars represent SNe-Ia data obtained from Ref. [5]. The two dashed curves represent the values (a) $m_h=6.40\times10^{-33}$ eV (lower dashed curve) and (b) $m_h=7.25\times10^{-33}$ eV (upper dashed curve). The solid curve represents the intermediate value (c) $m_h=6.93\times10^{-33}$ eV.

and a value of ht_0 in the range

$$8.10\ \text{Gyr}<ht_0<12.2\ \text{Gyr}.\qquad(41)$$

Estimates of h give $0.55<h<0.75$ [2]. For $h=0.65$, Eq. (41) gives a range for the age of the universe, namely

$$12.5\ \text{Gyr}<t_0<18.8\ \text{Gyr}.\qquad(42)$$

A representative solid curve (c) has the value $m_h=6.93\times10^{-33}$ eV, which gives $\Omega_0=0.346$ and $t_0=14.8$ Gyr (with $h=0.65$). A plot of the scale factor $a(t)$ for this curve is shown in Fig. 4.

V. RECOMBINATION, NUCLEOSYNTHESIS AND CMBR FLUCTUATIONS

In this section, we first calculate the time of recombination in our model, compare it to the recombination time in a standard open universe, and discuss the implications for big-bang nucleosynthesis. We then show that the apparent angular size of CMBR fluctuations in our model is somewhat smaller than in a spatially flat $\Omega_0=1$ model, although consistent with available data.

In our model, we find that recombination occurs in the matter-dominated era. The redshift at recombination is given by the following expression, discussed in Ref. [12]:
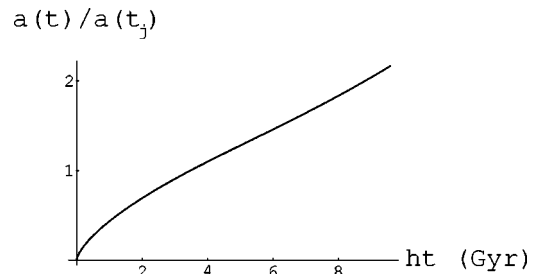

FIG. 4. A plot of $a(t)/a(t_j)$ versus ht in our model universe, for the value $m_h=6.93\times10^{-33}$ eV. The graph terminates at the x-coordinate value ht_0, t_0 being the present age of the universe.

$$z_r = 1048[1 + 0.0124(\Omega_b h^2)^{-0.738}][1 + g_1(\Omega_0 h^2)^{g_2}], \tag{43}$$

where

$$g_1 = 0.0783(\Omega_b h^2)^{-0.238}[1 + 39.5(\Omega_b h^2)^{0.763}]^{-1}, \tag{44}$$

$$g_2 = 0.560[1 + 21.1(\Omega_b h^2)^{1.81}]^{-1}, \tag{45}$$

and Ω_b is the ratio of baryon density and critical density at the present time. For the ranges $0.0025 < \Omega_b h^2 < 0.25$, $0.15 < \Omega_0 < 0.60$ and $0.55 < h < 0.75$ [2], we find

$$1055.73 < z_r < 1301.80. \tag{46}$$

The redshift at matter-radiation equality, z_{eq}, is given by

$$1 + z_{eq} = \Omega_0 h^2 / (\Omega_r h^2), \tag{47}$$

where Ω_r is the ratio of radiation energy density to critical density at the present time. With a CMBR temperature of 2.726 K at the present time [13], we obtain $\Omega_r h^2 = 4.16 \times 10^{-5}$. Using the same range of values as above for Ω_0 and h, we find

$$1090.14 < z_{eq} < 8115.90. \tag{48}$$

Therefore, in our model, recombination occurs in the matter-dominated era.

We now invert Eqs. (18) and (21) to obtain a range for the time of recombination, t_r, corresponding to the range of z_r in Eq. (46). A lower bound on t_r is obtained with the higher value of z_r and the lowest value of m_h consistent with a fit to the SNe-Ia data, i.e., $m_h = 6.40 \times 10^{-33}$ eV, and an upper bound is obtained with the lower value of z_r and the highest value of m_h consistent with the same data, i.e., $m_h = 7.25 \times 10^{-33}$ eV. We find

$$1.82 \times 10^5 \text{ yr} < h t_r < 4.97 \times 10^5 \text{ yr}. \tag{49}$$

We compare the above range with the corresponding range of $h t_r$ in a standard, spatially open matter-dominated model, with the same ranges of Ω_0, h and $\Omega_b h^2$ (and therefore the same range of z_r). The scale factor for an open matter-dominated model [11] leads to the parametrized equations

$$1 + z_r = \frac{2(1 - \Omega_0)}{\Omega_0}(\cosh \psi_r - 1)^{-1},$$

$$h t_r = (4.89 \times 10^9 \text{ yr}) \frac{\Omega_0}{(1 - \Omega_0)^{3/2}}(\sinh \psi_r - \psi_r), \tag{50}$$

ψ_r being a dimensionless parameter. The above equations give the following range for $h t_r$ in an open matter-dominated universe:

$$1.79 \times 10^5 \text{ yr} < h t_r < 4.89 \times 10^5 \text{ yr}. \tag{51}$$

Comparison of Eqs. (49) and (51) reveals that the ranges of recombination times for the two models differ by only

about 1.7%. Furthermore, the scale factors prior to recombination are almost identical in both models (the effect of spatial curvature in the open model is negligible for times prior to the recombination time). Therefore, it is expected that nuclear reaction calculations in big-bang nucleosynthesis would give nearly identical results for the abundances of light elements in both models. Since these calculations are known to give results in agreement with observation for the standard open model, nucleosynthesis calculations in our model would also give results that agree with observations, for the same values of Ω_0, h and $\Omega_b h^2$.

We now turn to the calculation of the apparent angular size of a fluctuation of a fixed proper size D at the surface of last scattering. This apparent angular size in our model will be compared to the apparent angular size of the same fluctuation in a spatially flat FRW model having $\Omega_0 = 1$, since the latter model is known to yield a CMBR fluctuation spectrum consistent with data.

For simplicity, we take the representative values $m_h = 6.93 \times 10^{-33}$ eV, $h = 0.65$, and $\Omega_b h^2 = 0.025$ in the discussion that follows. The chosen value of m_h corresponds to the intermediate curve (c) of Fig. 3, and gives $\Omega_0 = 0.346$. Equation (43) then gives the redshift at last scattering, as $z_r = 1089$.

The apparent angular size θ of a fluctuation of fixed proper size D at last scattering is given by

$$\theta = D/[a(t_r)r_1] = D(1 + z_r)^2/d_{L1}(z_r), \tag{52}$$

where $d_{L1}(z_r)$ is found from Eq. (28), which also gives

$$h d_{L1}(1089) = 1.017 \times 10^7 \text{ Mpc}. \tag{53}$$

Therefore, with $h = 0.65$,

$$\theta = (7.594 \times 10^{-2} \text{ Mpc}^{-1})D \tag{54}$$

in our model.

In a spatially flat $\Omega_0 = 1$ model, taking $h = 0.65$ and $\Omega_b h^2 = 0.025$, Eq. (43) gives $z_r = 1105$.

The luminosity distance d_L for a spatially flat $\Omega_0 = 1$ cosmology is given by

$$h d_L(z) = (5995.8 \text{ Mpc})(1 + z)[1 - (1 + z)^{-1/2}], \tag{55}$$

which gives

$$h d_L(1105) = 6.433 \times 10^6 \text{ Mpc}. \tag{56}$$

Therefore, with $h = 0.65$, and using Eq. (52), we obtain

$$\theta = (1.236 \times 10^{-1} \text{ Mpc}^{-1})D. \tag{57}$$

Comparison of Eqs. (54) and (57) reveals that the apparent angular size of a fluctuation of a given proper size at last scattering is about 1.6 times less in our model than in a spatially flat $\Omega_0 = 1$ model. If the first acoustic peak in the small angular scale CMBR spectrum arises from a fluctua-

tion of fixed proper size in all models, then the above result implies that, in our model, this peak would be shifted to a higher mode number relative to that in a $\Omega_0 = 1$ spatially flat model. In the latter model, the first peak is known to occur at mode number $l \approx 200$, and therefore it would occur at $l \approx 300$ in our model. These two possibilities are both consistent with the existing data on small angular scale CMBR fluctuations (see, for example, [2] and references therein).

VI. THE QUESTION OF FINE-TUNING

Recent observations of type-Ia supernovae evidently indicate that the universe has been in an accelerating phase from a redshift of order 1 up to the present time, which implies that the contribution of matter to the total energy density is of the same order of magnitude as the contribution of vacuum energy density at the present time [in spatially flat models, this means that $\Omega_0/(1-\Omega_0)$ is of order 1]. Why should this be the case? In mixed matter and cosmological constant models, this question requires an explanation of why the cosmological constant must be fine-tuned to a precise, non-zero value. As pointed out in [4], our model is relatively insensitive to the value of the cosmological constant term. However, in our model, it would appear that, within its allowed range of Eq. (37), the parameter m_h must be finely tuned to give values for z_j and $\Omega_0/(1-\Omega_0)$ that are within an order of magnitude of 1. It should be noted that the allowed range of values of m_h given by Eq. (37) does not by itself constrain z_j and Ω_0. The lowest allowed value of m_h gives $z_j = 0$ and $\Omega_0 = 1$, and the highest allowed value gives $z_j = \infty$ and $\Omega_0 = 0$.

Here, we argue that, in our model, values within an order of magnitude of 1 for z_j and $\Omega_0/(1-\Omega_0)$ are more likely than other values. The argument that follows rests on two assumptions: (i) $t_0 > t_j$ and (ii) all values of m_h within the allowed range given by Eq. (37) have equal *a priori* probability. Assumption (i) implies the range of m_h values of Eq. (37), and assumption (ii) is reasonable in lieu of a detailed fundamental theory that predicts the value of m_h.

Given these two assumptions, one may compute the probability distributions for z_j and Ω_0 in a straightforward manner, since both quantities are functions of m_h alone. By assumption (ii), the probability distribution function for m_h, $P(m_h)$, has the form

$$P(m_h) = P_0, \quad 3.69 \times 10^{-33} \text{ eV} < m_h < 7.39 \times 10^{-33} \text{ eV},$$

$$= 0, \quad \text{otherwise.} \tag{58}$$

Normalization of $P(m_h)$ yields $P_0 = 2.70 \times 10^{32}/\text{eV}$. We then obtain the probability distribution functions $P(z_j)$ and $P(\Omega_0/(1-\Omega_0))$ for z_j and $\Omega_0/(1-\Omega_0)$ respectively as

$$P(z_j) = P_0 \left/ \left| \frac{dz_j}{dm_h} \right| \right., \tag{59}$$

$$P(\Omega_0/(1-\Omega_0)) = P_0 \left/ \left| \frac{d(\Omega_0/(1-\Omega_0))}{dm_h} \right| \right., \tag{60}$$

where $z_j(m_h)$ and $\Omega_0(m_h)$ are given by Eqs. (31) and (33) respectively. We now compute the probability that z_j and $\Omega_0/(1-\Omega_0)$ lie between 0.1 and 10 (i.e., they are within an order of magnitude of 1). We find

$$P[0.1 < z_j < 10] = \int_{0.1}^{10} dz_j P(z_j) = 0.851,$$

$$P[0.1 < \Omega_0/(1-\Omega_0) < 10] = \int_{0.1}^{10} d\left(\frac{\Omega_0}{1-\Omega_0}\right) P\left(\frac{\Omega_0}{1-\Omega_0}\right)$$
$$= 0.619. \tag{61}$$

It is therefore more likely for z_j and $\Omega_0/(1-\Omega_0)$ to be within an order of magnitude of 1 rather than outside that range, assuming that all allowed values of m_h have equal *a priori* probability.

VII. CONCLUSIONS

We have shown that vacuum effects of a free scalar field of very low mass can account for the observed acceleration (i.e., the SNe-Ia data), while at the same time predicting reasonable values for the age of the universe and the total matter density. Evidently, our model also predicts reasonable light element abundances, and as a consequence of spatial flatness is in agreement with current data on small angular scale CMBR fluctuations.

Better SNe-Ia data would be able to distinguish between our model and mixed matter–cosmological-constant models, as well as quintessence models [3]. The curves of $\Delta(m-M)$ vs z are different in our model from those of the other models. Future observations of small angular size CMBR fluctuations may also distinguish between these models.

We emphasize that our model is based on a free renormalizable quantum field and does not require that we live at a very special time in the evolution of the universe. We also note that a graviton field of very low mass may give rise to vacuum effects similar to those of the scalar field we considered here. In contrast, a scalar particle of very high mass with similar characteristics to the present one may contribute to a stage of early inflation, with reheating and exit from inflation caused by particle production from the vacuum and possibly from other inflationary exit mechanisms.

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